Solutions

Math 45 6.7 Modeling and Solving Problems with Quadratic Equations

Objectives:

- Word Problems (Applications)
- Pythagorean Theorem

1) A ball is thrown off a cliff from a height of 240 feet above sea level. The height h (in feet) of the ball above the water at any time t (in seconds) can be modeled by the equation $h = -16t^2 + 32t + 240$

a. When will the height of the ball be 240 feet above sea level?

Substitute $h = 240$: $240 = -16t^2 + 32t + 240$

Set $= 0$: $0 = -16t^2 + 32t$

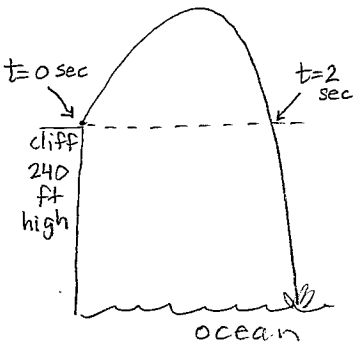
Factor: $0 = -16t(t-2)$

set factors $= 0$: $-16t = 0$ and $t - 2 = 0$

Solve: $\frac{-16t}{-16} = \frac{0}{-16}$

$t = 0$ sec

$t = 2$ sec



b. When will the ball strike the water?

Substitute $h = 0$: $0 = -16t^2 + 32t + 240$

GCF: $0 = -16(t^2 - 2t - 15)$

Factor: $0 = -16(t-5)(t+3)$

set factors $= 0$: $-16 \neq 0$ $t - 5 = 0$ $t + 3 = 0$

$t = 5$ sec $t \neq -3$

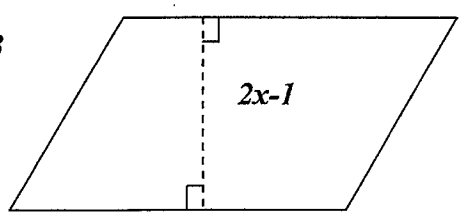
-15, -5, +3, -2

discard extraneous (negative) answer.

You'll need area formulas to do some of these problems.

Area of a Trapezoid:	Area of a Triangle:	Area of a Parallelogram
$A = \frac{1}{2}(B_1 + B_2)H$	$A = \frac{1}{2}BH$	$A = BH$

2) Use the given area to find the dimensions of the figure.



Write formula for area of parallelogram: $A = B \cdot H$

Substitute $A = 143$: $143 = B \cdot H$

Substitute $B = (2x+1)$ and $H = (2x-1)$: $143 = (2x+1)(2x-1)$

FOIL: $143 = 4x^2 - 2x + 2x - 1$

$143 = 4x^2 - 1$

Set $= 0$: $0 = 4x^2 - 144$

GCF: $0 = 4(x^2 - 36)$

Factor: $0 = 4(x-6)(x+6)$

Set Factors $= 0$: $4 \neq 0$ $x - 6 = 0$ $x + 6 = 0$

Solve: $x = 6$

Discard negative: $x \neq -6$

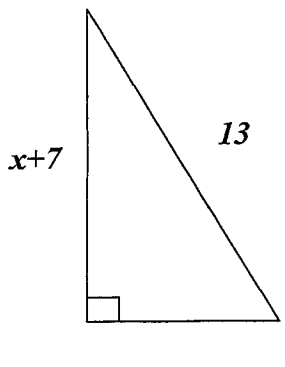
Substitute $x = 6$ to find B & H:

$B = 2(6) + 1 = 13$ cm $= B$

$H = 2(6) - 1 = 11$ cm $= H$

Math 45 6.7 Modeling and Solving Problems with Quadratic Equations p.2

3) Find the lengths of the sides of the triangle:



Write Pythagorean Theorem: $a^2 + b^2 = c^2$

Substitute $c = 13$. for hypotenuse. $a^2 + b^2 = 13^2$

Substitute $a = x$ and $b = (x+7)$ $x^2 + (x+7)^2 = 169$

FOIL $x^2 + x^2 + 14x + 49 = 169$

$2x^2 + 14x + 49 = 169$

Set = 0 $\frac{2x^2}{2} + \frac{14x}{2} - \frac{120}{2} = \frac{0}{2}$

GCF $x^2 + 7x - 60 = 0$

Factor $(x+12)(x-5) = 0$

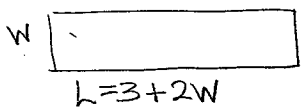
Set factors = 0 $x + 12 = 0$ $x - 5 = 0$

$x = -12$ $\boxed{x = 5}$

Discard negative length.

Substitute back $x + 7 = 5 + 7 = \boxed{12}$

4) The length of a rectangular hallway is 3 feet more than twice the width. If the area of the hallway is 44 square feet, what are the dimensions of the hallway.



Write formula for area of rectangle: $A = L \cdot W$

Substitute $A = 44$: $44 = L \cdot W$

Subst $L = (3+2W)$ $44 = (3+2W) \cdot W$

distribute $44 = 3W + 2W^2$

Set = 0 $0 = 2W^2 + 3W - 44$

Factor $0 = 2W^2 + 11W - 8W - 44$

$0 = W(2W+11) - 4(2W+11)$

$0 = (2W+11)(W-4)$

Set factors = 0 & solve $2W + 11 = 0$ $W - 4 = 0$

Discard negative. $2W = -11$ $\boxed{W = 4 \text{ ft}}$

Solve for $L = 3 + 2(4) = \boxed{11 \text{ ft} = L}$ $W = -11/2$

5) The height of a triangle is 5 inches less than the length of the base, and the area of the triangle is 42 square inches. Find the height of the triangle.

Write formula for area of triangle: $A = \frac{1}{2}BH$

Substitute $A = 42$ $42 = \frac{1}{2}BH$

Direct translation: $H = B - 5$

Substitute $42 = \frac{1}{2}B(B-5)$

clear fractions $84 = B(B-5)$

distribute $84 = B^2 - 5B$

Set = 0 $0 = B^2 - 5B - 84$

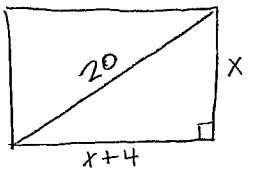
factor $0 = (B-12)(B+7)$

Set factors = 0. and solve $B - 12 = 0$ $B + 7 = 0$

discard negative $B = 12$ $B = -7$

Substitute to find height $H = B - 5$ $H = 12 - 5$ $\boxed{H = 7 \text{ in}}$

6) A rectangular 20-inch TV screen is 4 inches wider than it is tall. The size of TV screens is the diagonal measurement. Will this TV fit in your new media cabinet that is 20 inches wide?



Write Pythagorean Theorem: $a^2+b^2=c^2$

Substitute $c=20$: $a^2+b^2=20^2$

Substitute $a=x$ and $b=(x+4)$: $x^2+(x+4)^2=400$

FOIL: $x^2+(x+4)(x+4)=400$

$x^2+x^2+8x+16=400$

$2x^2+8x+16=400$

Set = 0: $\frac{2x^2}{2}+\frac{8x}{2}-\frac{384}{2}=\frac{0}{2}$

$x^2+4x-192=0$

Factor (-192: -12, 16; $-12+16=4$)

Set factors = 0: $(x-12)(x+16)=0$

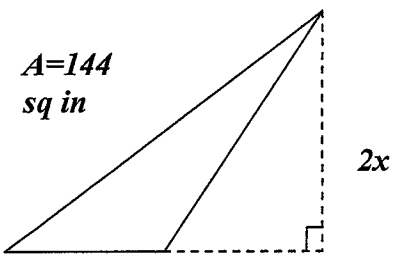
discard negative: $x-12=0$ $\quad x+16=0$

$x=12$ in $\quad x\neq -16$

12 inches wide is less than 20 inches wide

YES, TV will fit

7) Use the given area to find the dimensions of the figure.



Write formula for area of triangle: $A=\frac{1}{2}B\cdot H$

Subst $A=144$: $144=\frac{1}{2}B\cdot H$

Subst $B=(3x-6)$ and $H=2x$: $144=\frac{1}{2}(3x-6)\cdot 2x$

Simplify $\frac{1}{2}\cdot 2$: $144=x(3x-6)$

Dist: $144=3x^2-6x$

Set = 0: $\frac{0}{3}=\frac{3x^2}{3}-\frac{6x}{3}-\frac{144}{3}$

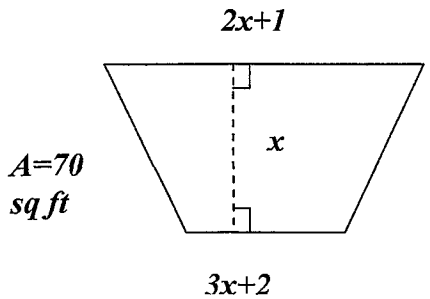
Factor (-48: -8, $+6$; $-8+6=-2$): $0=x^2-2x-48$

$(x-8)(x+6)=0$

$x-8=0$ $\quad x+6=0$

$x=8$ in $\quad x\neq -6$

8) Use the given area to find the dimensions of the figure.



Write formula for area of trapezoid: $A=\frac{1}{2}(B_1+B_2)H$

Subst $A=70$: $70=\frac{1}{2}(B_1+B_2)H$

Subst $B_1=2x+1$, $B_2=3x+2$, $H=x$: $70=\frac{1}{2}(2x+1+3x+2)\cdot x$

combine: $70=\frac{1}{2}(5x+3)x$

clear fraction: $140=x(5x+3)$

distribute: $140=5x^2+3x$

subtract 140: $0=5x^2+3x-140$

Factor (-700: -25, 28; $-25+28=3$): $0=5x^2-25x+28x-140$

Set factors = 0: $0=5x(x-5)+28(x-5)$

discard extraneous: $0=(x-5)(5x+28)$

subst into $2x+1$ and $3x+2$ and x.

$x-5=0$ $\quad 5x+28=0$

$x=5$ $\quad x\neq -\frac{28}{5}$

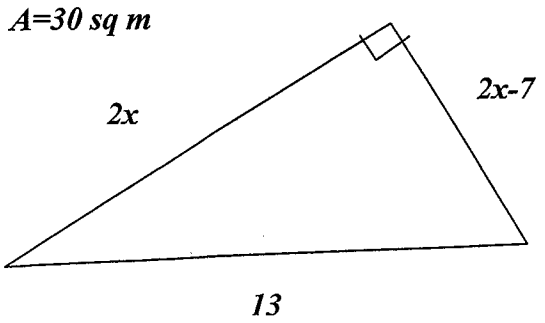
$2(5)+1=11$ ft $=B_1$

$3(5)+2=17$ ft $=B_2$

$5=H$

Math 45 6.7 Modeling and Solving Problems with Quadratic Equations p.4

9) Use the given area to find the dimensions of the figure.



Write the formula for the area of a triangle.

$A = \frac{1}{2}BH$

Subst $A=30$: $30 = \frac{1}{2}BH$

subst $\begin{cases} B=(2x-7) \\ H=2x \end{cases}$ $30 = \frac{1}{2}(2x-7)(2x)$

clear fractions $60 = 2x(2x-7)$

distribute $60 = 4x^2 - 14x$

set=0 $\frac{0}{2} = \frac{4x^2}{2} - \frac{14x}{2} - \frac{60}{2}$

factor $0 = 2x^2 - 7x - 30$

$0 = 2x^2 - 12x + 5x - 30$

$0 = 2x(x-6) + 5(x-6)$

$0 = (x-6)(2x+5)$

-60, -12, 5, -7

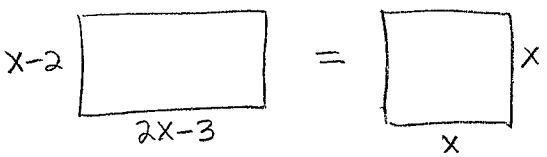
Set factors = 0 $x-6=0$ $2x+5=0$

discard negative $x=6$ $x \neq -\frac{5}{2}$

Substitute into $2x-7$ → $2(6)-7 = \boxed{5\text{ m}}$

$2x$ → $2(6) = \boxed{12\text{m}}$

10) A rectangle and a square have the same area. The width of the rectangle is 2 inches less than the side of the square and the length of the rectangle is 3 inches less than twice the side of the square. What are the dimensions of the rectangle?



Directly translate.

Write area of square = area rectangle

$x^2 = (x-2)(2x-3)$

FOIL $x^2 = 2x^2 - 3x - 4x + 6$

$x^2 = 2x^2 - 7x + 6$

Set = 0 $0 = x^2 - 7x + 6$

6, -6, -1, -7

factor $0 = (x-6)(x-1)$

set factor=0 $x=6$ $x=1$

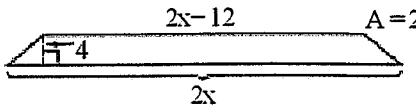
subst back $x-2$ $6-2=4$ $1-2=-1$ discard neg.

$2x-3$ $2(6)-3=9$

Rectangle is $\boxed{4\text{ in} \times 9\text{ in}}$

Square is 6 in x 6 in.

6.7.17 **Use the given area to find the dimensions of the quadrilateral.**



Need a space after the comma

The lengths of the two bases are 57,45.

(Type an integer or a simplified fraction. Use a comma to separate answers as needed.)

YOU ANSWERED: 32,20

Area of a trapezoid $A = \frac{1}{2}(B_1 + B_2)\cdot H$

Subst
- $A = 204$ (given)
- $B_1 = 2x$ (given)
- $B_2 = 2x-12$ (given)
- $H = 4$ (given)

$$204 = \frac{1}{2}(2x + 2x - 12)\cdot 4$$

Solve for x

$$204 = \frac{1}{2}\cdot 4(4x-12)$$
$$204 = 2(4x-12)$$
$$102 = 4x - 12$$
$$\frac{114}{4} = \frac{4x}{4}$$
$$28.5 = x.$$

Question asks for lengths of bases:

$2x \Rightarrow 2(28.5) = 57$

$2x-12 \Rightarrow 57-12 = 45.$

6.7.37 Fredo is an avid football fan and has purchased a plasma TV just in time to watch the big game. The TV screen is 17 inches taller than it is wide and there is a $1\frac{1}{2}$-inch-wide casing that surrounds the TV screen. Use this information to answer parts **(a)** and **(b)**.

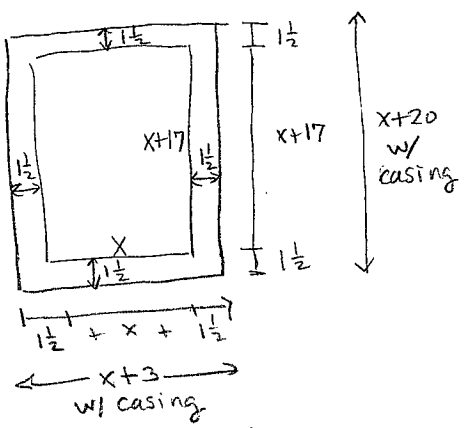
(a) Fredo begins to install the TV and remembers that he was told that it measures 53 inches on the diagonal, including the casing. What are the dimensions of the TV screen?

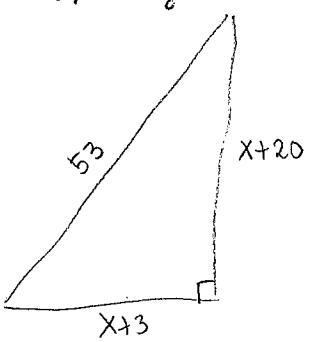
The width of the TV screen is ☐ inches.
(Type an integer or a simplified fraction.)

The height of the TV screen is ☐ inches.
(Type an integer or a simplified fraction.)

(b) What size opening is required to fit the TV into Fredo's entertainment center? (Enter the smaller dimension first).

The opening must be ☐ inches by ☐ inches.
(Type integers or simplified fractions.)





Weird! Taller than it is wide!!

a) Pythagorean Theorem $a^2+b^2=c^2$

$(x+3)^2 + (x+20)^2 = 53^2$

$(x+3)(x+3) + (x+20)(x+20) = 2809$

$x^2+6x+9 + x^2+40x+400 = 2809$

$2x^2 + 46x + 409 = 2809$

$-2809 \quad -2809$

$\frac{2x^2}{2} + \frac{46x}{2} - \frac{2400}{2} = \frac{0}{2}$

$x^2 + 23x - 1200 = 0$

$(x+48)(x-25) = 0$

$x+48=0 \qquad x-25=0$

$x=\cancel{-}48 \qquad x=25$

-1200
48 × -25
23

10,120
12,100
15,80
20,60
30,40
25,48 ✓

Width of screen: $x+3 \Rightarrow 25+3 =$ **28 in**

Height of screen: $x+17 \Rightarrow 25+17 =$ **42 in**

b) opening must fit the casing:

width $28+3 = 31$ in

height $42+3 = 45$ in

31 in × 45 in